A second-order immersed boundary method with near-wall physics

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APS/DFD
Minneapolis, MN
November 22, 2009
Motivation

electrical cables

wildland-urban interface fires

4 km

GIS terrain data (10 m resolution)
Some Previous Works


Fadlun’s methods

\[
\frac{u_i^{k+1} - u_i^k}{\Delta t} = F_i + B_i
\]

\[
B_i = -F_i + \frac{u_i^b - u_i^k}{\Delta t}
\]

\[
B_i = -F_i + \frac{\bar{u}_i^b - u_i^k}{\Delta t}
\]
Werner Wengle wall model

\[ u^* = 2.4 \ln z^* + 5.2 \]

\[ u^* = A(z^*)^B \]

\[ z^* = 11.81 \]

\[ \bar{u} = \frac{1}{\Delta z} \int_0^{\Delta z} u(z) \, dz \]

\[ \tau_w = \bar{\rho} \left[ \frac{1 - B}{2} A^{1+B} \left( \frac{\mu}{\bar{\rho} \Delta z} \right)^{1+B} + \frac{1 + B}{A} \left( \frac{\mu}{\bar{\rho} \Delta z} \right)^B \right]^{\frac{2}{1+B}} \]

Description of our new method

\[ a_{ij} = e_i \cdot \bar{e}_j \]

\[ \bar{u}_k = a_{ik} u_i \]

\[ g_{k\ell} = a_{ik} a_{j\ell} g_{ij}, \quad g_{ij} = \frac{\partial u_i}{\partial x_j} \]
Boundary Layer Equations

\[
\frac{\partial \bar{u}}{\partial s} + \frac{\partial \bar{v}}{\partial n} = D
\]

\[
\frac{\partial \bar{u}}{\partial l} + \bar{u} \frac{\partial \bar{u}}{\partial s} + \bar{v} \frac{\partial \bar{u}}{\partial n} = -\frac{1}{\rho} \left( \frac{\partial p}{\partial s} + \frac{\partial \bar{\tau}_{sn}}{\partial n} \right)
\]

Discretization:

\[
\bar{v}^{k+1} = \bar{v}_w + \delta n \left( D^{k+1} - \frac{1}{2} \frac{\partial \bar{u}}{\partial s} \right)_{2\delta n}
\]

\[
\frac{d\bar{u}}{dt} = - \left[ \bar{u} \frac{1}{2} \frac{\partial \bar{u}}{\partial s} \right]_{2\delta n} + \bar{v}^{k+1} \frac{1}{2} \left( \frac{\partial \bar{u}}{\partial n} \right)_{2\delta n} + \frac{\partial \bar{u}}{\partial n} \right]
\]

\[
+ \frac{1}{\rho} \left( \frac{\partial p}{\partial s} + \bar{\tau}_{sn} \frac{1}{2\delta n} - \bar{\tau}_{sn} \right)
\]
ODE solution method

\[
\begin{align*}
\frac{\partial \tilde{u}}{\partial n} \bigg|_w &= \frac{4}{3} \left( \frac{\tilde{u} - \tilde{u}_w}{\delta n} \right) - \frac{1}{3} \frac{\partial \tilde{u}}{\partial n} \bigg|_{2\delta n} + \mathcal{O}(\delta n^2) \\
\bar{\tau}_{sn} \bigg|_w &= -(1 - \text{SF}) \mu \left( \frac{\tilde{u} - \tilde{u}_w}{\delta n} \right)
\end{align*}
\]

From Werner Wengle

\[
\frac{d\tilde{u}}{dt} = a\tilde{u} + b
\]

\[
\tilde{u}(t) = \frac{(a\tilde{u}_0 + b)e^{at} - b}{a}
\]

\[
\begin{array}{l}
\text{rho} = 1.2; \\
\text{mu} = 0.001; \\
\text{dn} = 0.1; \\
\text{u0} = 1; \\
\text{u_wall} = 0; \\
v = .5; \\
duds = -.1; \\
dudn = 1; \\
dpds = -1; \\
tau = -.2; \\
\text{SF} = -100;
\end{array}
\]
Convergence
Rotating Cylinder
Cliff Lee’s two seam fastball (Re = 200,000)
Acknowledgements

• Nuclear Regulatory Commission
• Forest Service
• Building and Fire Research Laboratory, NIST