A second-order immersed boundary method with near-wall physics

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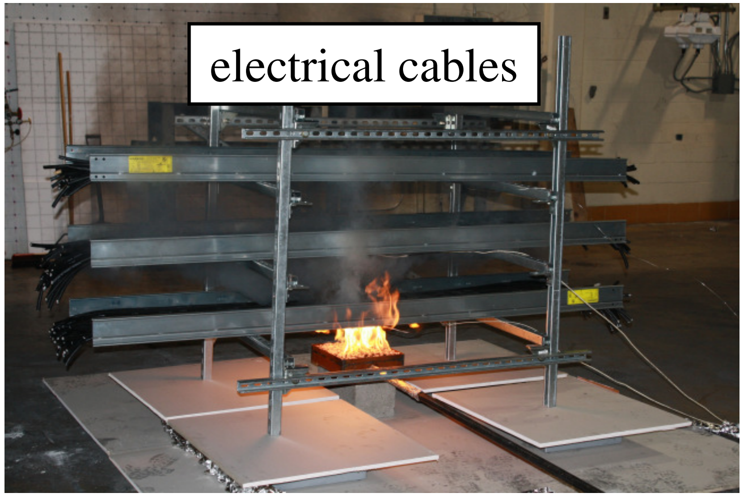
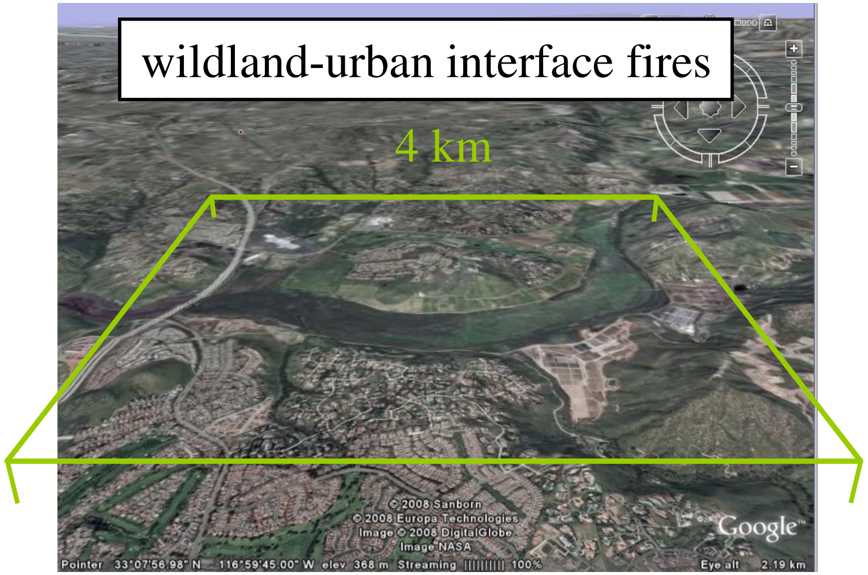
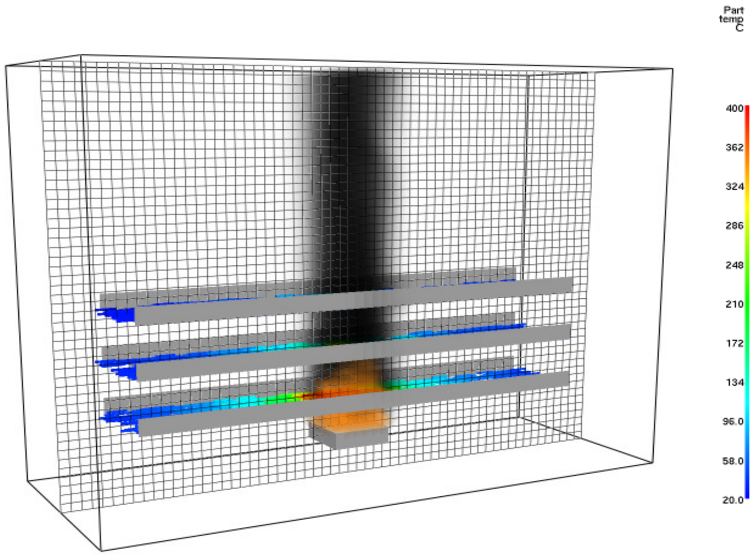
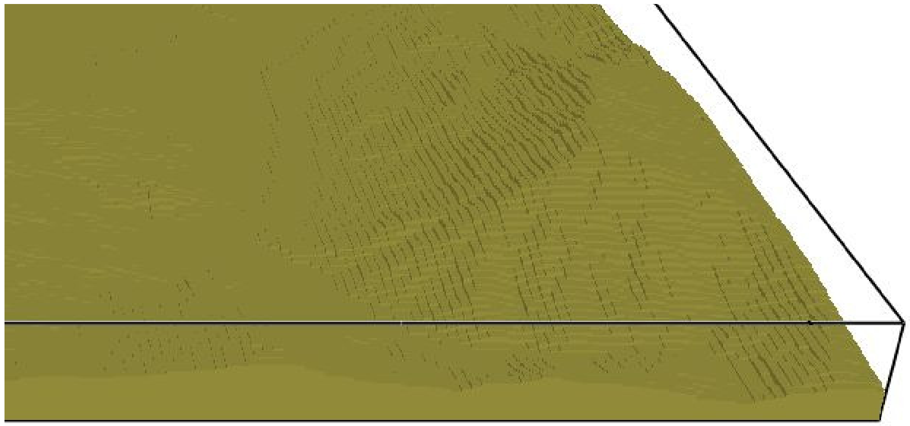
Motivation







GIS terrain data (10 m resolution)

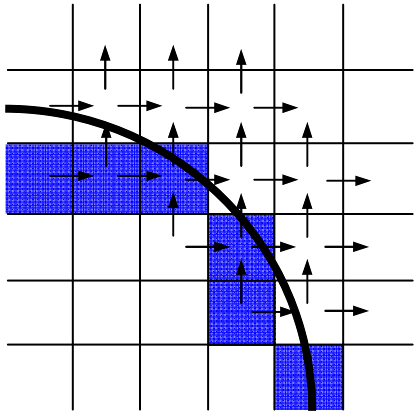


Some Previous Works

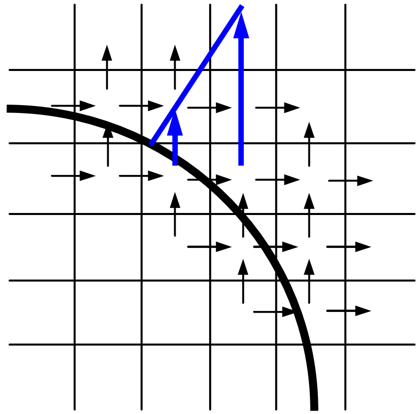
- M. J. B. M. Pourquie, ***Accuracy Close to the Wall for Large-Eddy Simulations of Flow Around Obstacles Using Immersed Boundary Methods***, In Quality and Reliability of Large-Eddy Simulations, J. Meyers, B. Geurts, and P. Sagaut, Eds., Springer, 2008.
- J. Emblemsvåg, R. Suzuki, and G. Candler, ***A Cartesian Grid Method for Moderate-Reynolds Number Flows around Complex and Moving Solid Objects***, AIAA Journal, 43(1):76-86, 2005.
- E. A. Fadlun, R. Verzicco, P. Orlandi, and J. Mohd-Yusof, ***Combined Immersed-Boundary Finite-Difference Methods for Three-Dimensional Complex Flow Simulations***, J. Comp. Phys., 161:35-60, 2000.

Fadlun's methods

$$\frac{u_i^{k+1} - u_i^k}{\Delta t} = F_i + B_i$$

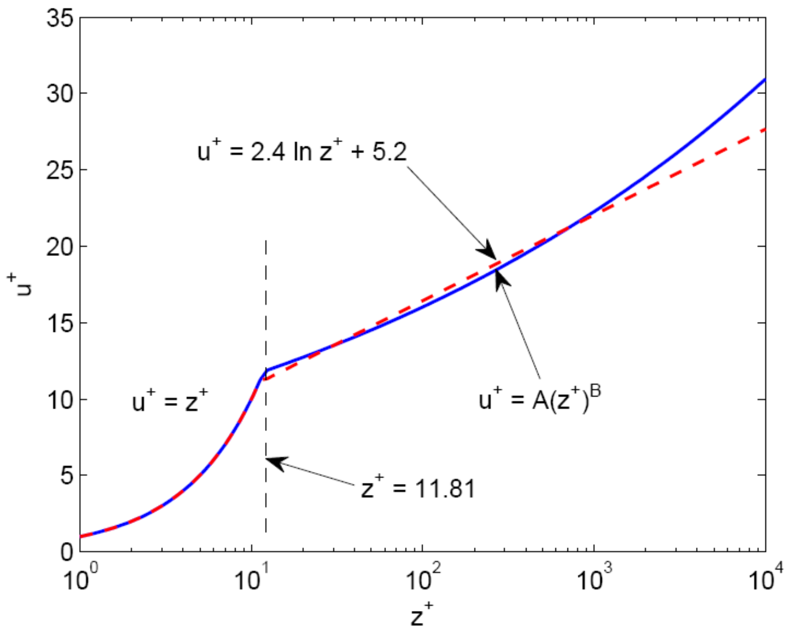


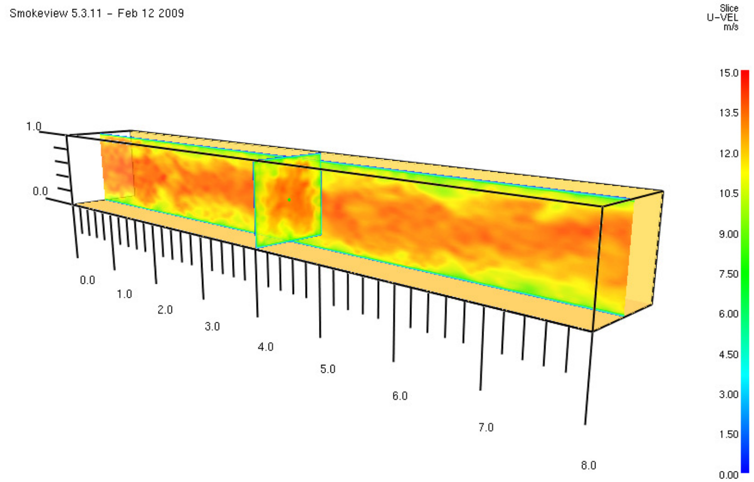
$$B_i = -F_i + \frac{u_i^b - u_i^k}{\Delta t}$$



$$B_i = -F_i + \frac{\overline{u}_i^b - u_i^k}{\Delta t}$$

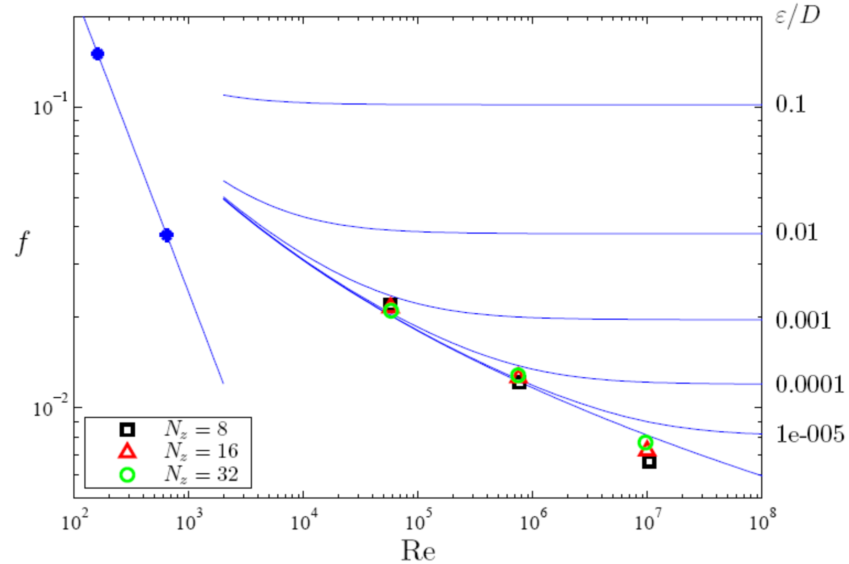
Werner Wengle wall model





$$\tilde{u} = \frac{1}{\Delta z}\int_0^{\Delta z} u(z)\,\mathrm{d}z$$

$$\tau_w = \bar{\rho}\left[\frac{1-B}{2}A^{\frac{1+B}{1-B}}\left(\frac{\mu}{\bar{\rho}\Delta z}\right)^{1+B} + \frac{1+B}{A}\left(\frac{\mu}{\bar{\rho}\Delta z}\right)^{B}\tilde{u}\right]^{\frac{2}{1+B}}$$



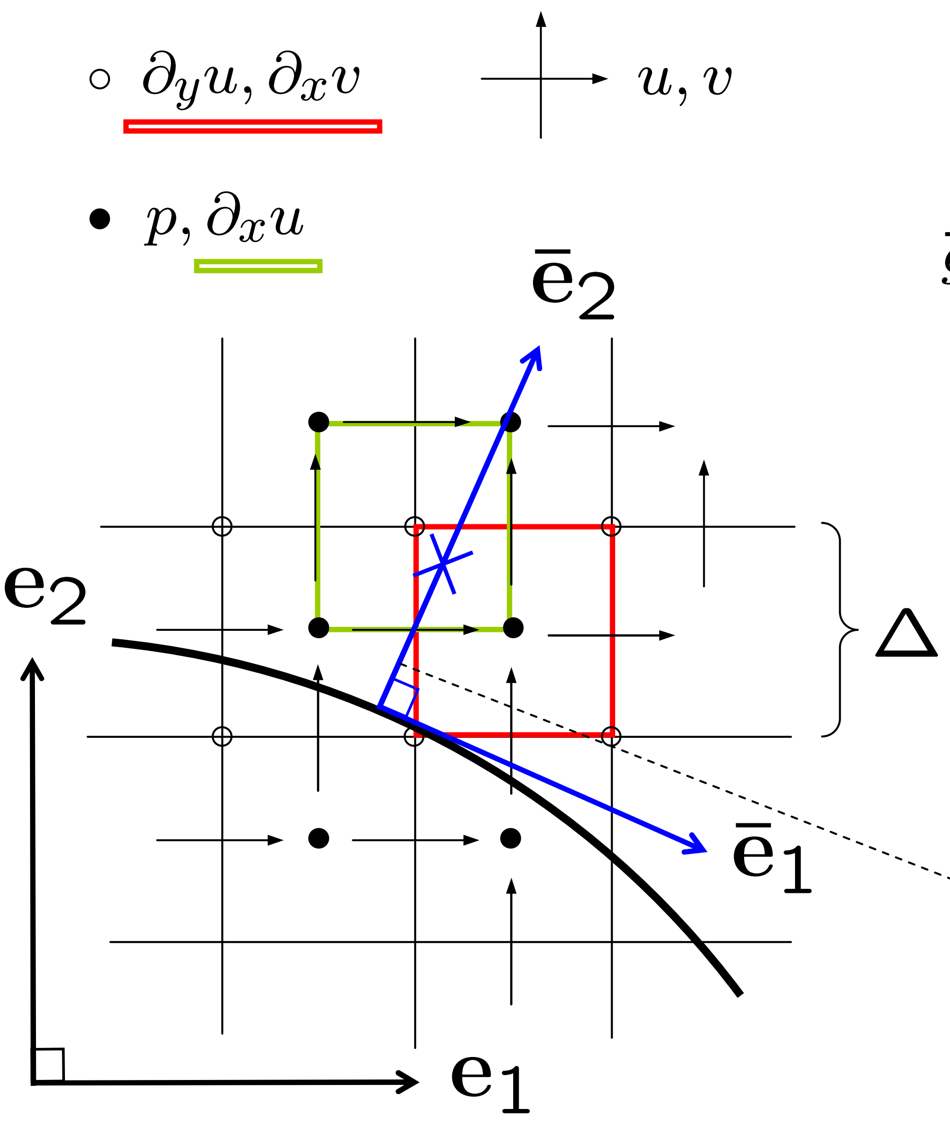
Werner, H., Wengle, H. (1991): Large-eddy simulation of turbulent flow over and around a cube in a plate channel. (8th Symposium on Turbulent Shear Flows, Munich, Germany).

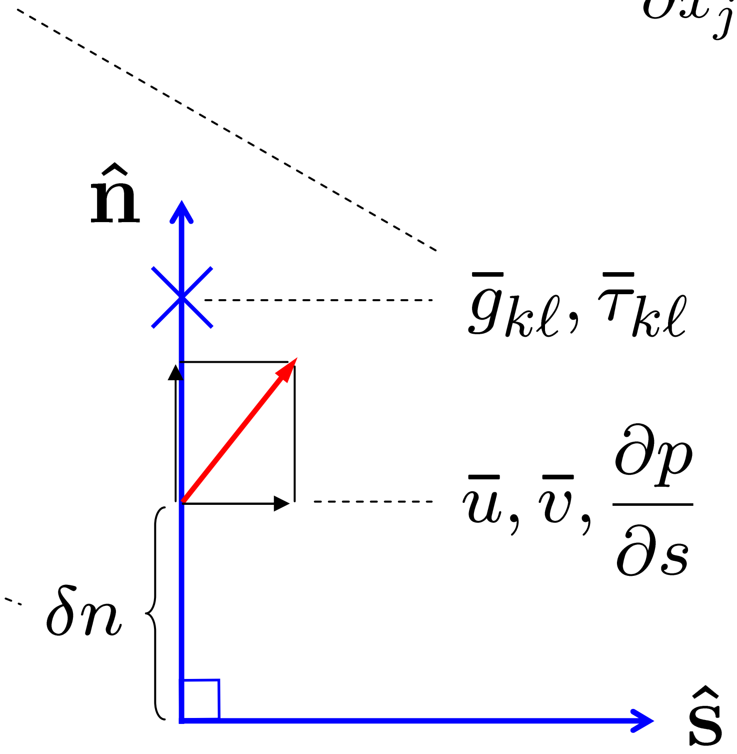
Description of our new method

$$a_{ij} = \mathbf{e}_i \cdot \bar{\mathbf{e}}_j$$

$$\bar{u}_k = a_{ik} u_i$$

$$\bar{g}_{k\ell} = a_{ik} a_{j\ell}\, g_{ij}, \qquad g_{ij} = \frac{\partial u_i}{\partial x_j}$$

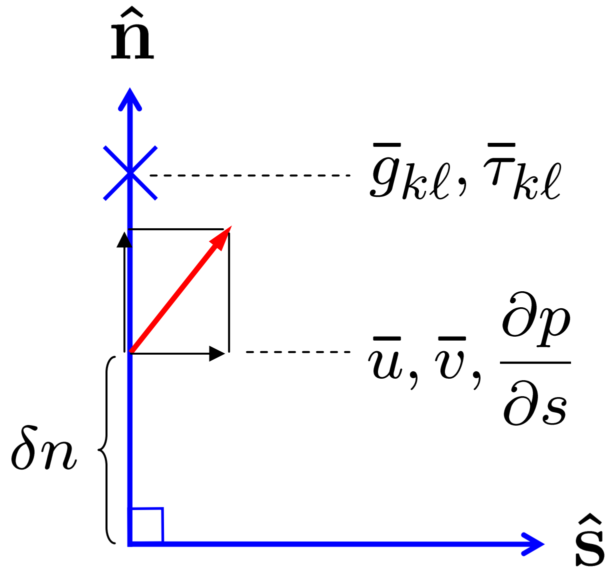




Boundary Layer Equations

$$\frac{\partial \bar{u}}{\partial s} + \frac{\partial \bar{v}}{\partial n} = D$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial s} + \bar{v}\frac{\partial \bar{u}}{\partial n} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial s} + \frac{\partial \bar{\tau}_{sn}}{\partial n}\right)$$



Discretization:

$$\bar{v}^{k+1} = \bar{v}_w + \delta n \left(D^{k+1} - \frac{1}{2}\left.\frac{\partial \bar{u}}{\partial s}\right|_{2\delta n}^{k}\right)$$

$$\frac{\mathrm{d}\bar{u}}{\mathrm{d}t} = -\left[\bar{u}\,\frac{1}{2}\left.\frac{\partial \bar{u}}{\partial s}\right|_{2\delta n}^{k} + \bar{v}^{k+1}\,\frac{1}{2}\left(\left.\frac{\partial \bar{u}}{\partial n}\right|_{2\delta n}^{k} + \left.\frac{\partial \bar{u}}{\partial n}\right|_{w}\right) + \frac{1}{\rho}\left(\left.\frac{\partial p}{\partial s}\right|_{n}^{k} + \frac{\bar{\tau}_{sn}|_{2\delta n}^{k} - \bar{\tau}_{sn}|_{w}}{2\delta n}\right)\right]$$

ODE solution method

$$\left.\frac{\partial \bar{u}}{\partial n}\right|_w = \frac{4}{3}\left(\frac{\bar{u}-\bar{u}_w}{\delta n}\right) - \frac{1}{3}\left.\frac{\partial \bar{u}}{\partial n}\right|_{2\delta n} + \mathcal{O}(\delta n^2)$$

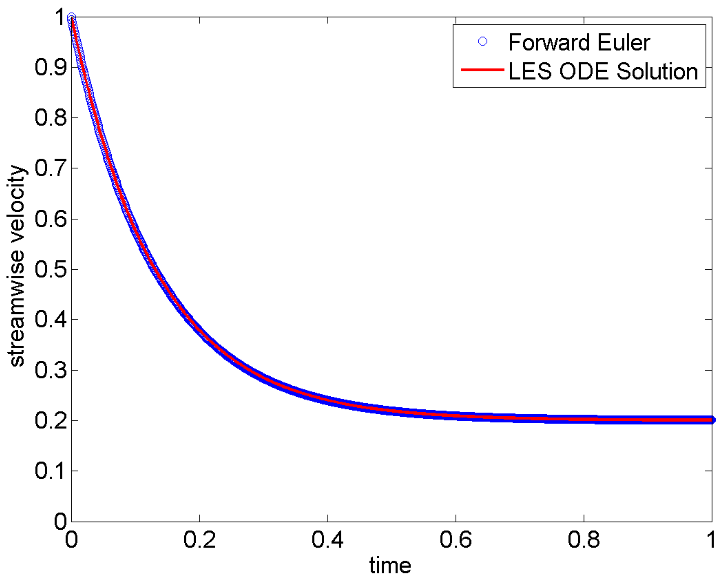
$$\left.\bar{\tau}_{sn}\right|_w = -(1-\mathtt{SF})\mu\left(\frac{\bar{u}-\bar{u}_w}{\delta n}\right)$$

From Werner Wengle

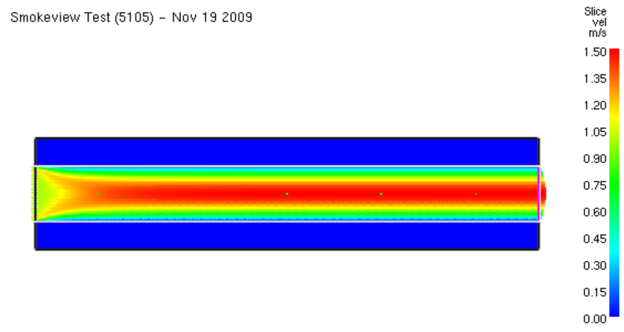
```
rho = 1.2;
mu = 0.001;
dn = 0.1;
u0 = 1;
u_wall = 0;
v = .5;
duds = -.1;
dudn = 1;
dpds = -1;
tau = -.2;
SF = -100;
```

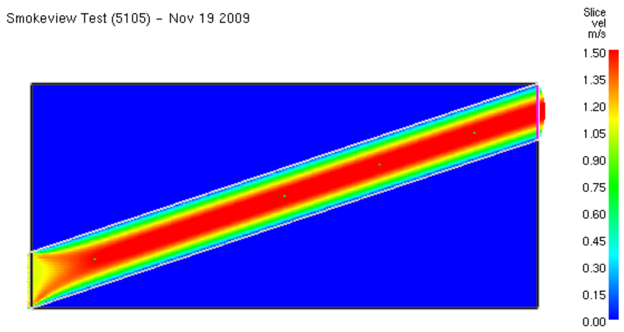
$$\frac{\mathrm{d}\bar{u}}{\mathrm{d}t} = a\bar{u} + b$$

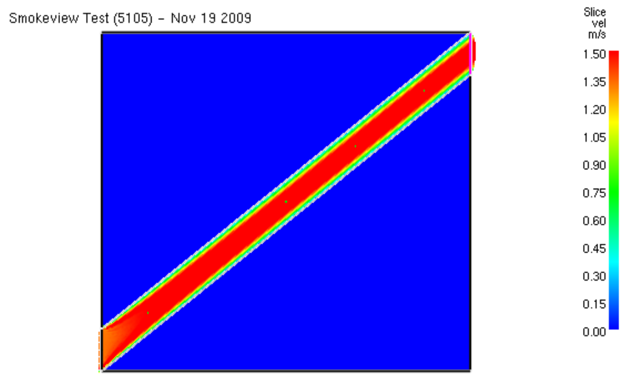
$$\bar{u}(t) = \frac{(a\bar{u}_0 + b)e^{at} - b}{a}$$

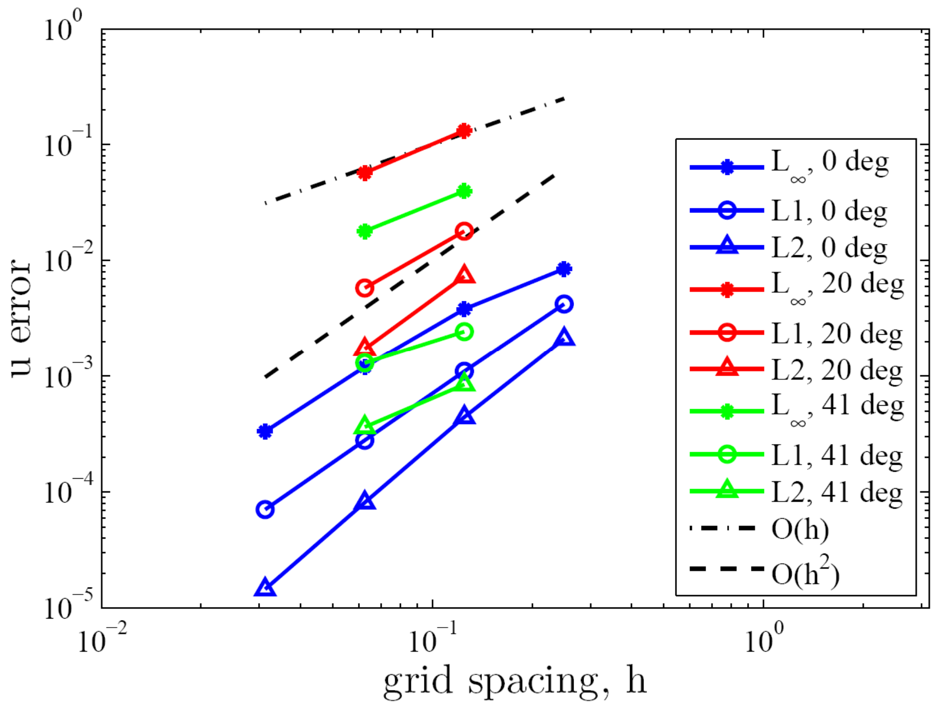


Convergence

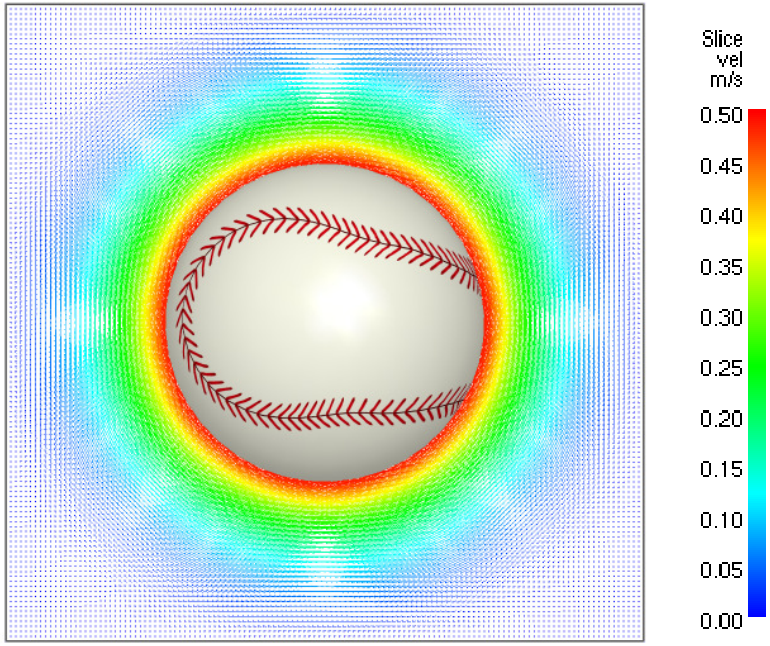




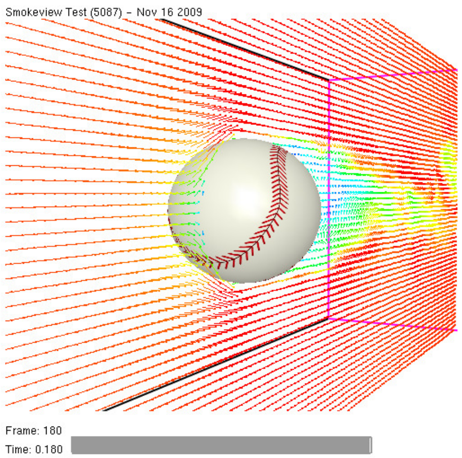


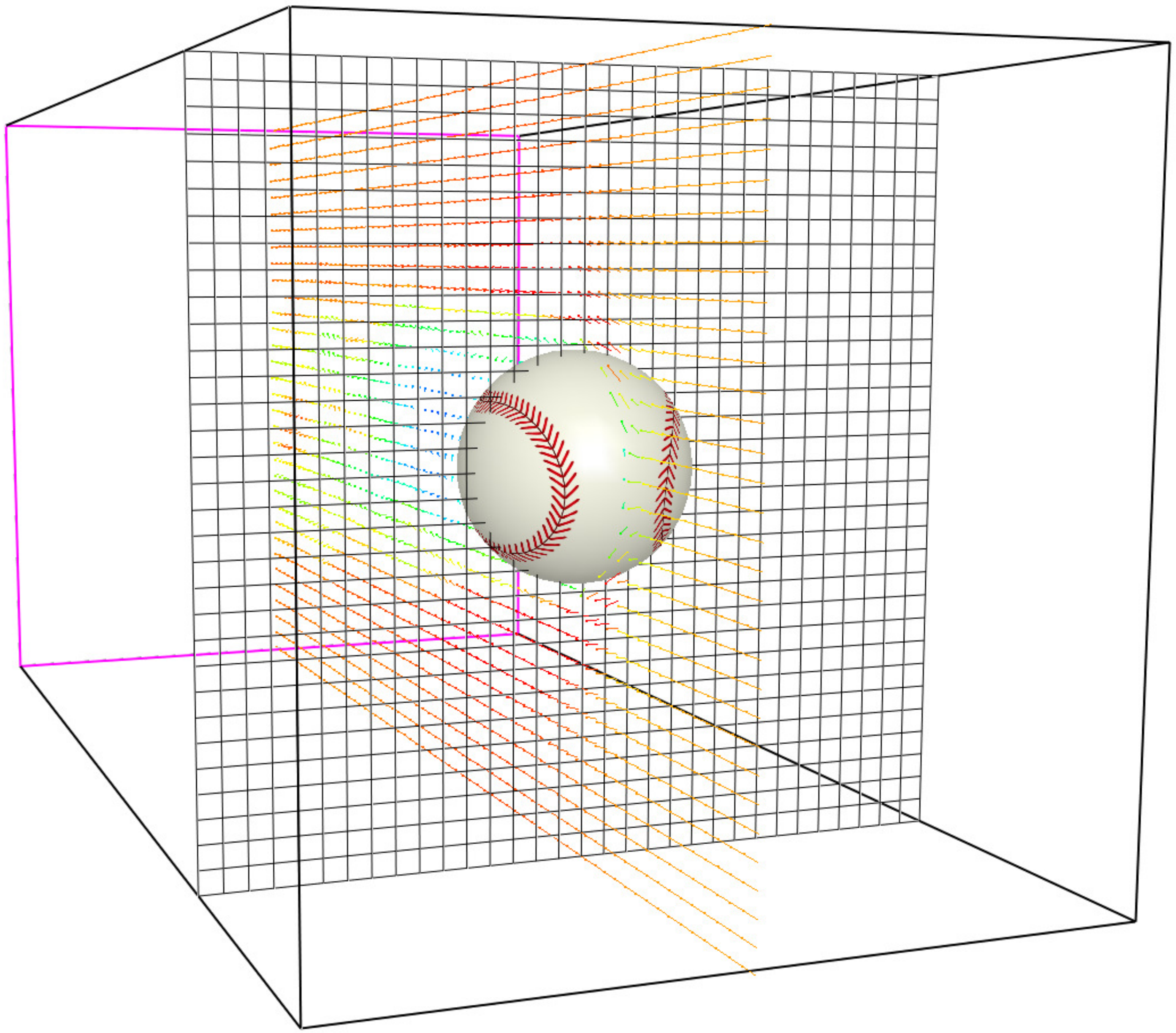


Rotating Cylinder



Cliff Lee’s two seam fastball (Re = 200,000)





Acknowledgements

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